Modeling of T/C complex stiffness modulus test and non-linearity of asphalt concrete mixes

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Abstract. The work described in this article is part of the French national project ANR MoveDVDC. One of the objectives of this project is to evaluate the influence of aging and damage of asphalt concrete materials on the lifetime of road pavements. In this context, modeling the behavior of aged or not asphalt mixes in fatigue is planned. At this moment, the first researches focused on the development of a phenomenological model reflecting the viscoelastic nature of asphalt mixtures to facilitate numerical modeling of structures. Indeed, the 2S2P1D rheological model can already correctly reproduce it, but the presence of fractional derivatives makes the modeling complicated. With this objective and relying on the observation of experimental T/C complex stiffness modulus test data, a "COULON model" with variable parameters was developed. It consists of two elements in parallel, which represent the real and imaginary parts of the complex stiffness modulus, such that their respective parameters named R_E and I_η vary according to temperature, pulsation and amplitude (for non-linearity) of sinusoidal loading. To allow a numerical implementation, the parameters R_E and I_η follow a CARREAU-YASUDA law as a function of the reduced pulsation, associating the effects of temperature and pulsation. In homogeneous conditions, this model can reproduce T/C complex stiffness modulus tests on cylindrical specimens.

Keywords: Bituminous mixtures, modeling, rheological and phenomenological models, complex stiffness modulus, non-linearity.

1 Introduction

One of the objectives of the French national project ANR MoveDVDC is to evaluate the influence of aging and damage of asphalt concrete materials on the lifetime of road pavements. In this context, modeling the behavior of aged or not asphalt mixes in fatigue is planned with the Discrete Element Method (DEM). It allows to model a test with a set of interconnected digital particles by a contact law.

The first researches, which are presented in this article, focused on the development of a contact law reflecting the viscoelastic nature of bituminous mixtures to facilitate numerical modeling of structures. To do this, experimental T/C complex stiffness modulus test data were used.

2 Viscoelasticity modeling: COULON model

2.1 Existing rheological models: advantages and disadvantages

The 2S2P1D model [1, 2, 3] can already correctly simulate the viscoelastic behavior of asphalt concrete mixes above the glass transition temperature T_g. But below T_g, modeling is approximate. For a bituminous mixture using a bitumen 35/50, T_g is around 30 °C. Despite this inconvenient, the 2S2P1D model stays very useful for precisely staking the complex stiffness modulus in COLE-COLE and BLACK spaces on a large temperature and frequency range by directly calculating it.

Unfortunately, the 2S2P1D model consists in two parabolic elements in series whose stress behavior is equal to a constant multiplied by the fractional derivative of the deformation. The presence of these fractional derivatives makes the modeling complicated for structural modelling with Finite Element Method (FEM) or DEM, but it remains popular for laboratory tests modeling.

The generalized MAXWELL and generalized KELVIN-VOIGT models are much more easily programmable. However, to correctly simulate the viscoelastic behavior of bituminous mixtures, a sufficient number of parameters is required. Thus, it is necessary to repeat at least ten to fifteen elements, in other words using at least twenty to thirty parameters. In comparison, the 2S2P1D model requires only nine parameters.

2.2 Proposal of a different design

Instead of trying to multiply the rheological elements, it might be useful to consider a phenomenological model for sinusoidal loadings representing directly the real and imaginary parts of the material complex stiffness modulus E_M^*, such that they can vary according to the entry conditions.

The proposal is therefore to use a simple model consisting of only two elements in parallel. The first one, of parameter R_E, represents the real part and is piloted by an elastic dominant. The second one, of parameter I_η, represents the imaginary part divided by the pulsation and is piloted by a viscous dominant. The components R_E and I_η can vary according to the temperature T of the environment, the pulsation ω of the imposed signal and the amplitude of deformation ε_0 of the imposed loading. Its code name is "COULON model", which can be abbreviated "CLN model" (Fig. 1).

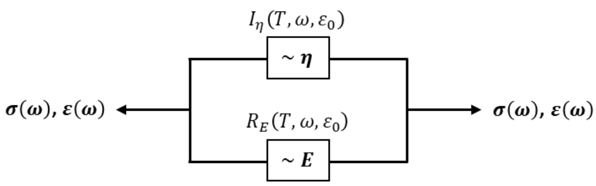

Fig. 1. Schematic diagram of the phenomenological COULON model.

3 Creation of the COULON(T,ω) model

3.1 Operating mode of the COULON(T,ω) model

For bitumen, temperature and frequency effects are dependent in linear viscoelasticity. This phenomenon is at the origin of the Time-Temperature Superposition Principle (TTSP). It is characterized using a translation factor a_T to calculate the reduced frequency ω_{R-T}, which connects these two physical quantities (Eq. (1)). The translation factor is calculated with the WILLIAMS-LANDEL-FERRY (WLF) relation according to the temperature T (Eq. (2)). T_{ref} is the chosen reference temperature, C_1 and C_2 are two constants.

$$\omega_{R-T} = a_T \cdot \omega \tag{1}$$

$$\log a_T = \frac{-C_1 \cdot (T - T_{ref})}{C_2 + (T - T_{ref})} \tag{2}$$

Variations of real part R_E and reduced imaginary part $I_{\eta,R-T}$ (Eq. (3)) as a function of the reduced pulsation can be fitted using CARREAU-YASUDA laws [4, 5]. Behavior of asphalt concrete mixes varies greatly around the glass transition temperature T_g. This implies to characterize a modeling below T_g (rubbery state) and a modeling above T_g (vitreous state) (Eqs. (4) and (5)). $R_{E,low}$, $I_{\eta,low}$ and $R_{E,up}$, $I_{\eta,up}$ are respectively lower and upper bounds, λ is a characteristic time, a et n are dimensionless parameters.

$$I_\eta = a_T \cdot I_{\eta,R-T} \tag{3}$$

$$R_E = \begin{cases} R_{E,low,1} \cdot \left[1 + \left[\lambda_{E,R-T,1} \cdot \omega_{R-T}\right]^{a_{E,1}}\right]^{\frac{n_{E,1}-1}{a_{E,1}}} & if\ \omega_{R-T} \leq \omega_{R-T_g} \\ R_{E,up,2} \cdot \left[1 + \left[\lambda_{E,R-T,2} \cdot \omega_{R-T}\right]^{a_{E,2}}\right]^{\frac{n_{E,2}-1}{a_{E,2}}} & if\ \omega_{R-T} \geq \omega_{R-T_g} \end{cases} \tag{4}$$

$a_{E,1} > 0$, $n_{E,1} > 1$, $a_{E,2} < 0$, $n_{E,2} > 1$, $n_{E,1} = n_{E,2}$, $\lambda_{E,R-T,1} = \left[{\omega_{R-T_g}}^2 \cdot \lambda_{E,R-T,2}\right]^{-1}$

$$I_{\eta,R-T} = \begin{cases} I_{\eta,R-T,low,1} \cdot \left[1 + \left[\lambda_{\eta,R-T,1} \cdot \omega_{R-T}\right]^{a_{\eta,1}}\right]^{\frac{n_{\eta,1}-1}{a_{\eta,1}}} & if\ \omega_{R-T} \leq \omega_{R-T_g} \\ I_{\eta,R-T,up,2} \cdot \left[1 + \left[\lambda_{\eta,R-T,2} \cdot \omega_{R-T}\right]^{a_{\eta,2}}\right]^{\frac{n_{\eta,2}-1}{a_{\eta,2}}} & if\ \omega_{R-T} \geq \omega_{R-T_g} \end{cases} \tag{5}$$

$a_{\eta,1} < 0$, $n_{\eta,1} < 1$, $a_{\eta,2} > 0$, $n_{\eta,2} < 1$, $\lambda_{\eta,R-T,1} = \left[{\omega_{R-T_g}}^2 \cdot \lambda_{\eta,R-T,2}\right]^{-1}$

Knowing R_E and I_η, the COULON model can be exploited by calculating the complex stiffness modulus E_M^* (Eq. (6)) and its derivatives: modulus norm, phase angle, ...

$$E_M^* = \Re(E_M^*) + i \cdot \Im(E_M^*) = R_E + i \cdot \omega \cdot I_\eta \tag{6}$$

3.2 Example with MANGIAFICO's experimental data (2014) [6]

In [6], T/C complex stiffness modulus tests were carried out by imposing a controlled sinusoidal deformation of amplitude 50 µm/m. The samples were cylindrical with a diameter of 75 mm and a height of 150 mm. The experimental data of the sample A.0.35-50.R4 were used here. The corresponding used bitumen binder has a grade 35/50 and composes 5.35% of the sample mass. The air gap is 3.7% of the volume. The experimental data were retrieved by graphical reading with the software "GetData Graph Digitizer".

The linear viscoelastic modeling parameters for the sample A.0.35-50.R4 are given in Tab. 1. The good fit of the model on the experimental data is observable in Fig. 2.

Table 1. A.0.35-50.R4 sample modeling parameters.

WLF law	$T_{ref} = 14.2\,°C$	$C_1 = 30.860$	$C_2 = 196.094\,°C$
Glass transition reduced pulsation		$\omega_{R-T_g} = 7.50 \cdot 10^{-4}\ rad/s$	
Real part $R_E(\omega_{R-T})$		**Imaginary part $I_{\eta,R-T}(\omega_{R-T})$**	
$\omega_{R-T} \leq \omega_{R-T_g}$	$\omega_{R-T} \geq \omega_{R-T_g}$	$\omega_{R-T} \leq \omega_{R-T_g}$	$\omega_{R-T} \geq \omega_{R-T_g}$
$R_{E,low,1} = 4.5\ MPa$	$R_{E,up,2} = 36500\ MPa$	$I_{\eta,R-T,low,1} = 3.00 \cdot 10^4\ MPa.s$	$I_{\eta,R-T,up,2} = 2.40 \cdot 10^6\ MPa.s$
$\lambda_{E,R-T,1} = 7.11 \cdot 10^1\ s/rad$	$\lambda_{E,R-T,2} = 2.50 \cdot 10^4\ s/rad$	$\lambda_{\eta,R-T,1} = 9.88 \cdot 10^3\ s/rad$	$\lambda_{\eta,R-T,2} = 1.80 \cdot 10^2\ s/rad$
$a_{E,1} = 0.220$	$a_{E,2} = -0.164$	$a_{\eta,1} = -0.206$	$a_{\eta,2} = 0.250$
$n_{E,1} = 2.850$	$n_{E,2} = 2.850$	$n_{\eta,1} = 0.090$	$n_{\eta,2} = -0.110$

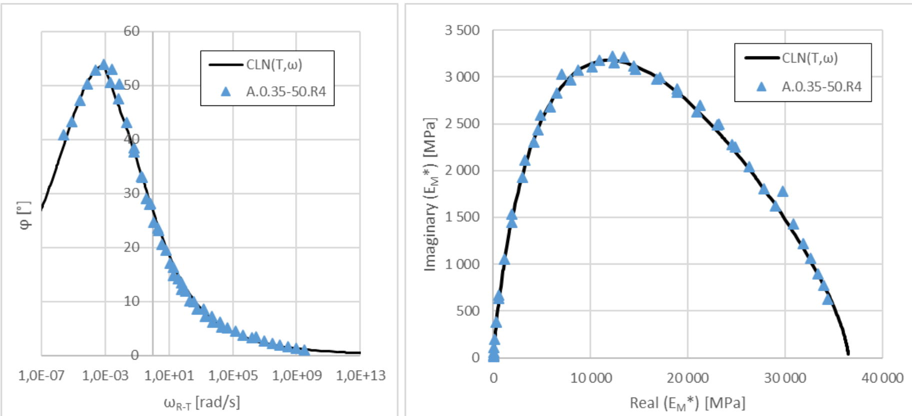

Fig. 2. Sample A.0.35-50.R4 (T/C complex stiffness modulus test, 50 µm/m) – Plot of experimental data and COULON model (T,ω) in the graph "phase angle φ function of reduced pulsation ω_{R-T}" (left) and in the "COLE-COLE space" (right). The maximum phase angle corresponds to the glass transition reduced pulsation ω_{R-T_g}.

4 Creation of the COULON(T,ω,ε₀) model

4.1 Operating mode of the COULON(T,ω,ε₀) model

It consists in applying the Time-Temperature-Amplitude Semi-Superposition Principle (TTASSP). First, the previous COULON(T,ω) model must be fitting to a reference amplitude $\varepsilon_{0,ref}$ that we choose. Then, this model is evolved in applying a new translation factor a_A to the reduced pulsation ω_{R-T} (Eq. (7)) and a power coefficient b_A to the upper bound $I_{\eta,R-T,up,2}$ (Eq. (8)). It has been observed that a_A and b_A seem following a law of type WLF according to ε_0 (Eqs. (9) and (10)). Careful, these observations were done for the only experimental data found in literature for three complete COLE-COLE curves (part 4.2) and are valid only above glass temperature. Further data will be needed later to confirm or evolve these non-linearity observations.

$$\omega_{R-TA} = a_A \cdot \omega_{R-T} \tag{7}$$

$$I_{\eta,R-TA,up,2} = I_{\eta,R-T,up,2}{}^{b_A} \tag{8}$$

$$\log a_A = \frac{-A_1 \cdot (\varepsilon_0 - \varepsilon_{0,ref})}{A_2 + (\varepsilon_0 - \varepsilon_{0,ref})} \tag{9}$$

$$\log b_A = \frac{-B_1 \cdot (\varepsilon_0 - \varepsilon_{0,ref})}{B_2 + (\varepsilon_0 - \varepsilon_{0,ref})} \tag{10}$$

4.2 Example with GRAZIANI's experimental data (2019) [7]

For this study in [7], a bituminous mixture for wearing course with a bitumen binder 70/100 dosed to 5.3% of the total mass is used. T/C complex stiffness modulus tests were carried out for three different deformation amplitudes: 15, 30 and 60 μm/m. The samples were cylindrical, D-94 mm x H-120 mm. The experimental data of the sample S2 are used here, which have an air void content of 8.5% of the total volume.

The non-linear viscoelastic modeling parameters for the sample S2 are given in Tab. 2. Fig. 3, on the left, shows the good fit of the model. On the right, we observe the non-linearity direction in the COLE-COLE space for different temperatures and frequencies.

Table 2. S2 sample modeling parameters.

WLF law for a_T	$T_{ref} = 14.2\,°C$	$C_1 = 30.860$	$C_2 = 196.094\,°C$
WLF law for a_A	$\varepsilon_{0,ref} = 30\ \mu m/m$	$A_1 = 1.020$	$A_2 = 90\ \mu m/m$
WLF law for b_A	$\varepsilon_{0,ref} = 30\ \mu m/m$	$B_1 = -0.0042$	$B_2 = 48\ \mu m/m$
Glass transition reduced pulsation for $\varepsilon_{0,ref}$		$\omega_{R-T_g,ref} = 6.10 \cdot 10^{-3}\ rad/s$	
Real part $R_E(\omega_{R-T})$ for $\omega_{R-T} \geq \omega_{R-T_g}$		**Im. part $I_{\eta,R-T}(\omega_{R-T})$ for $\omega_{R-T} \geq \omega_{R-T_g}$**	
$R_{E,up,2} = 27000\ MPa$		$I_{\eta,R-T,up,2} = 2.63 \cdot 10^6\ MPa.s$	
$\lambda_{E,R-T,2} = 4.00 \cdot 10^3\ s/rad$		$\lambda_{\eta,R-T,2} = 3.30 \cdot 10^2\ s/rad$	
$a_{E,2} = -0.164$ and $n_{E,2} = 2.250$		$a_{\eta,2} = 0.260$ and $n_{\eta,2} = -0.085$	

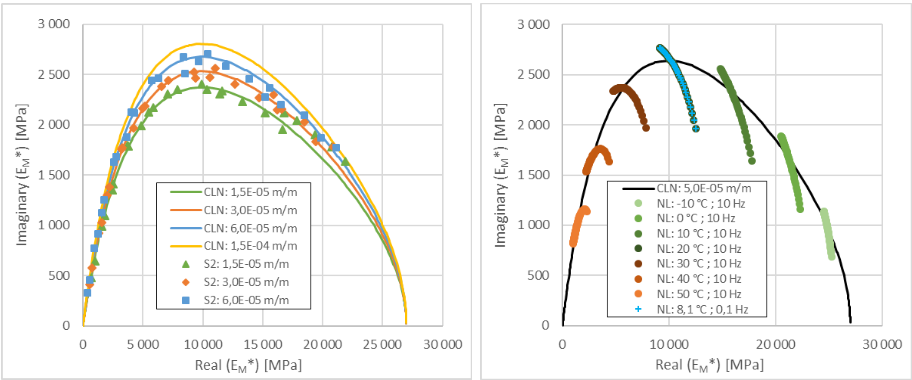

Fig. 3. Sample S2 (T/C complex stiffness modulus test, 15-30-60 µm/m) – On the left, plot of experimental data and COULON model (T,ω,ε$_0$) above the glass transition reduced pulsation ω_{R-T_g} in the COLE-COLE space. On the right, overlay in the COLE-COLE space of the non-linearity tests (0 to 100 µm/m over 50 cycles, for different temperatures at 10 Hz) and the complex stiffness modulus (50 µm/m) with the use of the COULON(T,ω,ε$_0$) model. To verify the TTSP, one test is done to (20 °C, 10 Hz) and another one to (8.1 °C, 0.1 Hz).

5 Conclusion

The CLN(T,ω) and (T,ω,ε$_0$) models fit the experimental data accurately without using fractional derivatives. Ten parameters are needed for the CLN(T,ω) model for $\omega_{R-T} \geq \omega_{R-T_g}$, sixteen for the complete CLN(T,ω) model and fourteen for the CLN(T,ω,ε$_0$) model with $\omega_{R-T} \geq \omega_{R-T_g}$. Thus, depending on situation, the model is adaptable.

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